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Author(s)	Suzuki, Tomonari
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NUMBERS ON DIAMETER IN ν-GENERALIZED METRIC SPACES

Tomonari Suzuki

Abstract

We study some numbers on diameter in ν-generalized metric spaces.

1. Introduction

Throughout this paper, we define the meaning of "$\{x_n\}_{n=1}^{\mu}\neq$" by that $\{x_n\}_{n=1}^{\mu}$ is a finite sequence and $x_1, x_2, \ldots, x_\mu$ are all different. We denote by $\mathbf{N}$ the set of all positive integers.

In 2000, Branciari introduced the following very interesting concept.

Definition 1 (Branciari [2]). Let X be a set, let d be a function from $X \times X$ into $[0, \infty)$ and let $\nu \in \mathbf{N}$. Then (X, d) is said to be a *ν-generalized metric space* if the following hold:

(N1) $d(x, y) = 0$ iff $x = y$ for any $x, y \in X$.
(N2) $d(x, y) = d(y, x)$ for any $x, y \in X$.
(N3) $d(x, y) \le D(x, u_1, u_2, \ldots, u_\nu, y)$ for any $x, u_1, u_2, \ldots, u_\nu, y \in X$ such that $x, u_1, u_2, \ldots, u_\nu, y$ are all different, where $D(x, u_1, u_2, \ldots, u_\nu, y) = d(x, u_1) + d(u_1, u_2) + \cdots + d(u_\nu, y)$.

It is obvious that (X, d) is a metric space if and only if (X, d) is a 1-generalized metric space. We found that not every generalized metric space has the compatible topology. See Example 7 in [3] and Example 4.2 in [6]. In [1] and [7], we discussed the completeness and compactness of ν-generalized metric spaces, respectively. See also [5].

In [4], we obtain the following lemmas:

Lemma 2 ([4]). *Let ν be an odd positive integer and let $\mu \in \mathbf{N}$ with $\mu \ge \nu + 3$. Then there exists a positive integer M satisfying the following:*

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- *If (X, d) is a ν-generalized metric space, ε is a positive real number and $\{x_j\}_{j=1}^{\mu} {}^{\neq}$ is a finite sequence in X such that $d(x_j, x_{j+1}) \le \varepsilon$ for any $j = 1, 2, \dots, \mu - 1$, then*

$$d(x_i, x_j) \le M\varepsilon$$

holds for any $i, j \in \{1, 2, \dots, \mu\}$.

Lemma 3 ([4]). *Let ν be an even positive integer and let $\mu \in \mathbf{N}$ with $\mu \ge \nu + 3$. Then there exists a positive integer M satisfying the following:*

- *If (X, d) is a ν-generalized metric space, ε is a positive real number and $\{x_j\}_{j=1}^{\mu} {}^{\neq}$ is a finite sequence in X such that $d(x_j, x_{j+1}) \le \varepsilon$ for any $j = 1, 2, \dots, \mu - 1$, then*

$$d(x_i, x_j) \le M\varepsilon$$

holds for any $i, j \in \{1, 2, \dots, \mu\}$ such that $i - j$ is odd.

We denote by $M(\nu, \mu)$ the minimum integer M in Lemmas 2 and 3. In this paper, we study $M(\nu, \mu)$.

2. Results

We begin with the following proposition, whose proof is obvious.

Proposition 4. *Let (X, d) and (X, e) be ν-generalized metric spaces, let S be a bijection on X and let c be a positive real number. Define function f_j from $X \times X$ into $[0, \infty)$ as follows:*

$$f_1(x, y) = d(Sx, Sy) \tag{1}$$

$$f_2(x, y) = cd(x, y) \tag{2}$$

$$f_3(x, y) = d(x, y) + e(x, y) \tag{3}$$

$$f_4(x, y) = \max\{d(x, y), e(x, y)\} \tag{4}$$

for any $x, y \in X$. Then (X, f_j) are ν-generalized metric spaces.

We first consider the case where ν is odd.

Lemma 5. *Let $\nu \in \mathbf{N}$ be odd and let $\mu \in \mathbf{N}$ satisfy $\mu \ge 2\nu + 1$. Put $X = \{0, 1, \dots, \mu - 1\}$ and let d be a function from $X \times X$ into $[0, \infty)$ such that (X, d) is a ν-generalized metric space and $d(i, i+1) \le 1$ holds for any $i \in \{0, 1, \dots, \mu - 2\}$. Then for any $j \in \mathbf{N} \cup \{0\}$, $k \in \mathbf{N}$ and $\ell \in \{1, 2, \dots, (\nu - 1)/2\} =: \Lambda$, the following hold:*

$$d(j, j + k\nu + 1) \le k\nu + 1 \tag{5}$$

$$d(j, j + 2\ell + 1) \le 2\nu + 1 \tag{6}$$

$$d(j, j + k\nu + 2\ell + 1) \le (k + 2)\nu + 1 \tag{7}$$

$$d(j, j + 2\ell) \le 3\nu + 1 \tag{8}$$

and

$$d(j, j + k\nu + 2\ell) \le (k + 1)\nu + 1 \tag{9}$$

provided the left hand side can be calculated.

Proof. Define a function d' from $X \times X$ into $[0, \infty)$ by

$$d'(i, j) = \max\{d(i, j), d(\mu - 1 - i, \mu - 1 - j)\}.$$

Then by Proposition 4 (1) and (4), (X, d') is a ν-generalized metric space. Since $d \le d'$, $d'(i, i + 1) \le 1$ and there appears no d in the right hand sides of (5)–(9), we write d instead of d' for our convenience. Then we have

$$d(i, j) = d(j, i) = d(\mu - 1 - i, \mu - 1 - j) = d(\mu - 1 - j, \mu - 1 - i). \tag{10}$$

In order to show (5), we note that (5) holds for $k = 0$. Fix j, k with $j + k\nu + 1 < \mu$ and assume that (5) holds for $k := k - 1$. Then we have

$$\begin{aligned} d(j, j + k\nu + 1) &\le D(j, j + (k - 1)\nu + 1, \dots, j + k\nu + 1) \\ &\le ((k - 1)\nu + 1) + \nu = k\nu + 1. \end{aligned}$$

By induction, we have shown (5). In the case of $\nu = 1$, we have $\Lambda = \varnothing$. Thus (6)–(9) holds. So we assume $\nu \ge 3$ from now on. In order to show (6), from (10), we may assume

$$(j - 1) - 0 + 1 \le (\mu - 1) - (j + 2\ell + 2) + 1.$$

and hence $2(j + \ell) \le \mu - 2$. So we have

$$j + \ell + \nu + 1 \le \mu/2 - 1 + \nu + 1 = \mu/2 + \nu \le \mu/2 + (\mu - 1)/2 < \mu.$$

Using this and (5), we have

$$\begin{aligned} d(j, j + 2\ell + 1) &\le D(j, \dots, j + \ell, j + \ell + \nu + 1, \dots, j + 2\ell + 1) \\ &\le \nu + (\nu + 1) = 2\nu + 1. \end{aligned}$$

Let us prove (7). We note that (7) becomes (6) in the case of $k = 0$. Fix j, k, ℓ with $j + k\nu + 2\ell + 1 < \mu$ and assume that (7) holds for $k := k - 1$. Then we have

$$\begin{aligned} d(j, j + k\nu + 2\ell + 1) &\le D(j, j + (k - 1)\nu + 2\ell + 1, \dots, j + k\nu + 2\ell + 1) \\ &\le ((k + 2 - 1)\nu + 1) + \nu = (k + 2)\nu + 1. \end{aligned}$$

By induction, we have shown (7). In order to show (8), from (10), we may assume

$$(j-1) - 0 + 1 \le (\mu - 1) - (j + 2\ell + 1) + 1.$$

and hence $2(j+\ell) \le \mu - 1$. So we have

$$j + \ell + v \le (\mu - 1)/2 + v \le \mu - 1 < \mu.$$

From (6), we have

$$d(j, j+v) \le 2v + 1.$$

Using this, we have

$$\begin{aligned} d(j, j+2\ell) &\le D(j, \dots, j+\ell, j+\ell+v, \dots, j+2\ell) \\ &\le v + (2v+1) = 3v + 1. \end{aligned}$$

In order to show (9), from (10), we may assume

$$(j-1) - 0 + 1 \le (\mu - 1) - (j + v + 2\ell + 1) + 1.$$

and hence $2(j+\ell) \le \mu - v - 1$. So we have

$$j + (v-1)/2 + \ell + v + 1 \le (\mu - v - 1)/2 + (v-1)/2 + v + 1 = \mu/2 + v < \mu.$$

Using this and (5), we have

$$\begin{aligned} &d(j, j+v+2\ell) \\ &\quad \le D(j, \dots, j + (v-1)/2 + \ell, j + (v-1)/2 + \ell + v + 1, \dots, j + v + 2\ell) \\ &\quad \le v + (v+1) = 2v + 1. \end{aligned}$$

Thus, (9) holds for $k = 1$. Fix j, k, ℓ with $j + kv + 2\ell < \mu$ and assume that (9) holds for $k := k - 1$. Then we have

$$\begin{aligned} d(j, j+kv+2\ell) &\le D(j, j + (k-1)v + 2\ell, \dots, j + kv + 2\ell) \\ &\le ((k+1-1)v + 1) + v = (k+1)v + 1. \end{aligned}$$

By induction, we have shown (9). □

Theorem 6. *Let $v \in \mathbf{N}$ be odd and let $\mu \in \mathbf{N}$ satisfy $\mu \ge 2v + 1$. Let $k \in \mathbf{N} \cup \{0\}$ satisfy*

$$(k-1)v + 4 \le \mu \le kv + 3.$$

Then

$$M(v, \mu) \le (k+1)v + 1.$$

Proof. Let (X, d) be a ν-generalized metric space and let ε be a positive real number and $\{x_j\}_{j=1}^{\mu}{}^{\neq}$ be a finite sequence in X such that $d(x_j, x_{j+1}) \le \varepsilon$ for any $j = 1, 2, \dots, \mu - 1$. Noting Proposition 4 (2), we may assume $\varepsilon = 1$. From now on, we write j instead of x_j. In the case where $\nu = 1$, we have

$$M(\nu, \mu) = \mu - 1 = k + 2 = (k+1)\nu + 1.$$

So the conclusion holds. In the other case, where $\nu \ge 3$, since $2\nu + 1 \le \mu \le k\nu + 3$, we have $2 - 2/\nu \le k$ and hence $2 \le k$. We put

$$M = (k+1)\nu + 1.$$

For $k' \in \mathbf{N}$ with $k' < k$ and $\ell \in \{1, 2, \dots, (\nu-1)/2\}$, we have by Lemma 5

$$d(j, j + k'\nu + 1) \le k'\nu + 1 < M$$

$$d(j, j + 2\ell + 1) \le 2\nu + 1 < M$$

$$d(j, j + k'\nu + 2\ell + 1) \le (k'+2)\nu + 1 \le M$$

$$d(j, j + 2\ell) \le 3\nu + 1 \le M$$

$$d(j, j + k'\nu + 2\ell) \le (k'+1)\nu + 1 < M$$

$$d(j, j + k\nu + 1) \le k\nu + 1 < M$$

$$d(j, j + k\nu + 2) \le (k+1)\nu + 1 = M$$

provided the left hand side can be calculated. Thus we obtain $d(i, j) \le M$ for any i, j. So the conclusion holds. $\square$

We next consider the other case, where ν is even.

Lemma 7. *Let $\nu \in \mathbf{N}$ be even and let $\mu \in \mathbf{N}$ satisfy $\mu \ge 2\nu + 1$. Put $X = \{0, 1, \dots, \mu - 1\}$ and let d be a function from $X \times X$ into $[0, \infty)$ such that (X, d) is a ν-generalized metric space and $d(i, i+1) \le 1$ holds for any $i \in \{0, 1, \dots, \mu - 2\}$. Then for any $j \in \mathbf{N} \cup \{0\}$, $k \in \mathbf{N}$ and $\ell \in \{1, 2, \dots, \nu/2 - 1\} =: \Lambda$, the following hold:*

$$d(j, j + k\nu + 1) \le k\nu + 1 \tag{11}$$

$$d(j, j + 2\ell + 1) \le 2\nu + 1 \tag{12}$$

and

$$d(j, j + k\nu + 2\ell + 1) \le (k+1)\nu + 1 \tag{13}$$

provided the left hand side can be calculated.

Remark. We cannot prove neither (8) nor (9).

PROOF. As in the proof of Lemma 5, we may assume (10) and we can prove (11) and (12). In order to show (13), from (10), we may assume

$$(j-1)-0+1 \leq (\mu-1)-(j+\nu+2\ell+2)+1.$$

and hence $2(j+\ell) \leq \mu-\nu-2$. So we have

$$j+\nu/2+\ell+\nu+1 \leq (\mu-\nu-2)/2+\nu/2+\nu+1 = \mu/2+\nu < \mu.$$

Using this and (11), we have

$$\begin{aligned} &d(j, j+\nu+2\ell+1) \\ &\quad \leq D(j, \dots, j+\nu/2+\ell, j+\nu/2+\ell+\nu+1, \dots, j+\nu+2\ell+1) \\ &\quad \leq \nu+(\nu+1) = 2\nu+1. \end{aligned}$$

Hence (13) holds for $k=1$. Fix j, k, ℓ with $j+k\nu+2\ell+1 < \mu$ and assume that (13) holds for $k := k-1$. Then we have

$$\begin{aligned} d(j, j+k\nu+2\ell+1) &\leq D(j, j+(k-1)\nu+2\ell+1, \dots, j+k\nu+2\ell+1) \\ &\leq ((k+1-1)\nu+1)+\nu = (k+1)\nu+1. \end{aligned}$$

By induction, we have shown (13). □

THEOREM 8. *Let $\nu \in \mathbf{N}$ be even and let $\mu \in \mathbf{N}$ satisfy $\mu \geq 2\nu+1$. Let $k \in \mathbf{N}$ satisfy*

$$(k-1)\nu+4 \leq \mu \leq k\nu+3.$$

Then

$$M(\nu, \mu) \leq k\nu+1.$$

PROOF. Let X, d and $\{j\}_{j=1}^{\mu}$ be as in the proof of Theorem 6. We put

$$M = k\nu+1.$$

In the case where $\nu=2$, we note $\{1, 2, \dots, \nu/2-1\} =: \Lambda$ is empty. For $k' \in \mathbf{N}$ with $k' < k$, we have by Lemma 7

$$\begin{aligned} d(j, j+k'\nu+1) &\leq k'\nu+1 < M \\ d(j, j+k\nu+1) &\leq k\nu+1 = M \end{aligned}$$

provided the left hand side can be calculated. Thus the conclusion holds. In the other case, where $\nu \geq 4$, we note $2 \leq k$. For $k' \in \mathbf{N}$ with $k' < k$ and $\ell \in \Lambda$, we have by Lemma 7

$$\begin{aligned} d(j, j+k'\nu+1) &\leq k'\nu+1 < M \\ d(j, j+2\ell+1) &\leq 2\nu+1 \leq M \end{aligned}$$

$$d(j, j + k'\nu + 2\ell + 1) \le (k' + 1)\nu + 1 \le M$$
$$d(j, j + k\nu + 1) \le k\nu + 1 = M$$

provided the left hand side can be calculated. Thus the conclusion holds. □

3. Conjectures

By computer, we can conjecture the following:

Conjecture 9. Let $\nu \in \mathbf{N}$ be odd and let $\mu \in \mathbf{N}$ satisfy $\mu \ge 2\nu + 1$. Put $X = \{0, 1, \dots, \mu - 1\}$ and define a function d from $X \times X$ into $[0, \infty)$ by

$$d(j, j) = 0$$
$$d(j, j + 1) = 1$$
$$d(j, j + k\nu + 1) = k\nu + 1$$
$$d(j, j + 2\ell + 1) = 2\nu + 1$$
$$d(j, j + k\nu + 2\ell + 1) = (k + 2)\nu + 1$$
$$d(j, j + 2\ell) = 3\nu + 1$$
$$d(j, j + k\nu + 2\ell) = (k + 1)\nu + 1$$

for any $j \in \mathbf{N} \cup \{0\}$, $k \in \mathbf{N}$ and $\ell \in \{1, 2, \dots, (\nu - 1)/2\}$, where the left hand side can be defined. Then (X, d) is a ν-generalized metric space.

Conjecture 10. Let $\nu \in \mathbf{N}$ be odd and let $\mu \in \mathbf{N}$ satisfy $\mu \ge 2\nu + 1$. Let $k \in \mathbf{N} \cup \{0\}$ satisfy $(k - 1)\nu + 4 \le \mu \le k\nu + 3$. Then $M(\nu, \mu) = (k + 1)\nu + 1$.

Conjecture 11. Let $\nu \in \mathbf{N}$ be even and let $\mu \in \mathbf{N}$ satisfy $\mu \ge 2\nu + 1$. Put $X = \{0, 1, \dots, \mu - 1\}$ and define a function d from $X \times X$ into $[0, \infty)$ by

$$d(j, j) = 0$$
$$d(j, j + 1) = 1$$
$$d(j, j + k\nu + 1) = k\nu + 1$$
$$d(j, j + 2\ell + 1) = 2\nu + 1$$
$$d(j, j + k\nu + 2\ell + 1) = (k + 1)\nu + 1$$
$$d(j, j + 2\ell) = \alpha$$
$$d(j, j + k\nu + 2\ell) = \alpha$$

$$d(j, j+\nu) = \alpha$$
$$d(j, j+k\nu) = \alpha$$

for any $j \in \mathbf{N} \cup \{0\}$, $k \in \mathbf{N}$ and $\ell \in \{1, 2, \ldots, \nu/2 - 1\}$, where the left hand side can be defined and α is a positive number which is large enough. Then (X, d) is a ν-generalized metric space.

Conjecture 12. Let $\nu \in \mathbf{N}$ be even and let $\mu \in \mathbf{N}$ satisfy $\mu \geq 2\nu + 1$. Let $k \in \mathbf{N}$ satisfy $(k-1)\nu + 4 \leq \mu \leq k\nu + 3$. Then $M(\nu, \mu) = k\nu + 1$.

By computer, we can conjecture the value of $M(\nu, \mu)$ with $\nu + 3 \leq \mu \leq 2\nu$.

Conjecture 13.

$M(3,6) = 13$			
$M(5,8) = 31$	$M(5,9) = 26$	$M(5,10) = 21$	
$M(7,10) = 57$	$M(7,11) = 36$	$\cdots$	$M(7,13) = 36$
$M(7,14) = 29$			
$M(9,12) = 91$	$M(9,13) = 64$	$M(9,14) = 46$	$\cdots$
$M(9,17) = 46$	$M(9,18) = 37$		
$M(11,14) = 133$	$M(11,15) = 100$	$M(11,16) = 78$	$M(11,17) = 56$
$\cdots$	$M(11,21) = 56$	$M(11,22) = 45$	
$M(13,16) = 183$	$M(13,17) = 118$	$M(13,18) = 92$	$M(13,19) = 92$
$M(13,20) = 66$	$\cdots$	$M(13,25) = 66$	$M(13,26) = 53$

Conjecture 14.

$M(4,7) = 13$	$M(4,8) = 13$		
$M(6,9) = 25$	$M(6,10) = 25$	$M(6,11) = 19$	$M(6,12) = 19$
$M(8,11) = 49$	$M(8,12) = 33$	$\cdots$	$M(8,14) = 33$
$M(8,15) = 25$	$M(8,16) = 25$		
$M(10,13) = 81$	$M(10,14) = 51$	$M(10,15) = 41$	$\cdots$
$M(10,17) = 41$	$M(10,18) = 31$	$\cdots$	$M(10,20) = 31$
$M(12,15) = 121$	$M(12,16) = 85$	$M(12,17) = 49$	$\cdots$
$M(12,20) = 49$	$M(12,21) = 37$	$\cdots$	$M(12,24) = 37$

$M(14,17) = 169$ $\quad$ $M(14,18) = 113$ $\quad$ $M(14,19) = 85$ $\quad$ $M(14,20) = 57$

$\cdots$ $\quad$ $M(14,24) = 57$ $\quad$ $M(14,25) = 43$ $\quad$ $\cdots$

$M(14,28) = 43$

4. Examples

We finally give examples of ν-generalized metric spaces which are strongly connected with this study. These examples are made by computer and α is a positive number which is large enough.

- $[\nu = 3,\ \mu = 5]$

$x\backslash y$	0	1	2	3	4
0	0	1	α	α	4
1	1	0	1	α	α
2	α	1	0	1	α
3	α	α	1	0	1
4	4	α	α	1	0

- $[\nu = 3,\ \mu = 6]$

$x\backslash y$	0	1	2	3	4	5
0	0	1	10	7	4	13
1	1	0	1	10	13	4
2	10	1	0	1	10	7
3	7	10	1	0	1	10
4	4	13	10	1	0	1
5	13	4	7	10	1	0

- $[\nu = 3,\ \mu = 10]$

$x\backslash y$	0	1	2	3	4	5	6	7	8	9
0	0	1	10	7	4	7	10	7	10	13
1	1	0	1	10	7	4	7	10	7	10
2	10	1	0	1	10	7	4	7	10	7
3	7	10	1	0	1	10	7	4	7	10
4	4	7	10	1	0	1	10	7	4	7
5	7	4	7	10	1	0	1	10	7	4
6	10	7	4	7	10	1	0	1	10	7
7	7	10	7	4	7	10	1	0	1	10
8	10	7	10	7	4	7	10	1	0	1
9	13	10	7	10	7	4	7	10	1	0

- $[\nu = 5,\ \mu = 8]$

$x\backslash y$	0	1	2	3	4	5	6	7
0	0	1	26	11	16	21	6	31
1	1	0	1	26	21	16	31	6
2	26	1	0	1	26	21	16	21
3	11	26	1	0	1	26	21	16
4	16	21	26	1	0	1	26	11
5	21	16	21	26	1	0	1	26
6	6	31	16	21	26	1	0	1
7	31	6	21	16	11	26	1	0

- $[\nu = 5,\ \mu = 9]$

$x\backslash y$	0	1	2	3	4	5	6	7	8
0	0	1	26	11	16	11	6	21	16
1	1	0	1	16	11	16	21	6	21
2	26	1	0	1	16	21	16	21	6
3	11	16	1	0	1	26	21	16	11
4	16	11	16	1	0	1	16	11	16
5	11	16	21	26	1	0	1	16	11
6	6	21	16	21	16	1	0	1	26
7	21	6	21	16	11	16	1	0	1
8	16	21	6	11	16	11	26	1	0

- $[\nu = 5,\ \mu = 10]$

$x\backslash y$	0	1	2	3	4	5	6	7	8	9
0	0	1	16	11	16	11	6	11	16	21
1	1	0	1	16	11	16	11	6	21	16
2	16	1	0	1	16	11	16	21	6	11
3	11	16	1	0	1	16	21	16	11	6
4	16	11	16	1	0	1	16	11	16	11
5	11	16	11	16	1	0	1	16	11	16
6	6	11	16	21	16	1	0	1	16	11
7	11	6	21	16	11	16	1	0	1	16
8	16	21	6	11	16	11	16	1	0	1
9	21	16	11	6	11	16	11	16	1	0

- $[\nu = 7,\ \mu = 10]$

$x\backslash y$	0	1	2	3	4	5	6	7	8	9
0	0	1	50	15	36	29	22	43	8	57
1	1	0	1	50	29	36	43	22	57	8
2	50	1	0	1	50	29	36	43	22	43
3	15	50	1	0	1	50	43	36	43	22
4	36	29	50	1	0	1	50	29	36	29
5	29	36	29	50	1	0	1	50	29	36
6	22	43	36	43	50	1	0	1	50	15
7	43	22	43	36	29	50	1	0	1	50
8	8	57	22	43	36	29	50	1	0	1
9	57	8	43	22	29	36	15	50	1	0

- $[\nu = 2,\ \mu = 10]$

$x\backslash y$	0	1	2	3	4	5	6	7	8	9
0	0	1	α	3	α	5	α	7	α	9
1	1	0	1	α	3	α	5	α	7	α
2	α	1	0	1	α	3	α	5	α	7
3	3	α	1	0	1	α	3	α	5	α
4	α	3	α	1	0	1	α	3	α	5
5	5	α	3	α	1	0	1	α	3	α
6	α	5	α	3	α	1	0	1	α	3
7	7	α	5	α	3	α	1	0	1	α
8	α	7	α	5	α	3	α	1	0	1
9	9	α	7	α	5	α	3	α	1	0

- $[\nu = 4,\ \mu = 7]$

$x\backslash y$	0	1	2	3	4	5	6
0	0	1	α	9	α	5	α
1	1	0	1	α	13	α	5
2	α	1	0	1	α	13	α
3	9	α	1	0	1	α	9
4	α	13	α	1	0	1	α
5	5	α	13	α	1	0	1
6	α	5	α	9	α	1	0

- $[\nu = 4,\ \mu = 8]$

$x\backslash y$	0	1	2	3	4	5	6	7
0	0	1	α	9	α	5	α	13
1	1	0	1	α	9	α	5	α
2	α	1	0	1	α	13	α	5
3	9	α	1	0	1	α	9	α
4	α	9	α	1	0	1	α	9
5	5	α	13	α	1	0	1	α
6	α	5	α	9	α	1	0	1
7	13	α	5	α	9	α	1	0

- $[\nu = 4,\ \mu = 10]$

$x\backslash y$	0	1	2	3	4	5	6	7	8	9
0	0	1	α	9	α	5	α	9	α	9
1	1	0	1	α	9	α	5	α	9	α
2	α	1	0	1	α	9	α	5	α	9
3	9	α	1	0	1	α	9	α	5	α
4	α	9	α	1	0	1	α	9	α	5
5	5	α	9	α	1	0	1	α	9	α
6	α	5	α	9	α	1	0	1	α	9
7	9	α	5	α	9	α	1	0	1	α
8	α	9	α	5	α	9	α	1	0	1
9	9	α	9	α	5	α	9	α	1	0

- $[\nu = 6,\ \mu = 9]$

$x\backslash y$	0	1	2	3	4	5	6	7	8
0	0	1	α	13	α	19	α	7	α
1	1	0	1	α	25	α	19	α	7
2	α	1	0	1	α	25	α	19	α
3	13	α	1	0	1	α	25	α	19
4	α	25	α	1	0	1	α	25	α
5	19	α	25	α	1	0	1	α	13
6	α	19	α	25	α	1	0	1	α
7	7	α	19	α	25	α	1	0	1
8	α	7	α	19	α	13	α	1	0

- $[\nu = 6,\ \mu = 10]$

$x\backslash y$	0	1	2	3	4	5	6	7	8	9
0	0	1	α	13	α	13	α	7	α	19
1	1	0	1	α	13	α	19	α	7	α
2	α	1	0	1	α	19	α	19	α	7
3	13	α	1	0	1	α	25	α	19	α
4	α	13	α	1	0	1	α	19	α	13
5	13	α	19	α	1	0	1	α	13	α
6	α	19	α	25	α	1	0	1	α	13
7	7	α	19	α	19	α	1	0	1	α
8	α	7	α	19	α	13	α	1	0	1
9	19	α	7	α	13	α	13	α	1	0

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Tomonari Suzuki
Department of Basic Sciences
Faculty of Engineering
Kyushu Institute of Technology
Tobata, Kitakyushu 804-8550*, Japan*
E-mail: suzuki-t@mns.kyutech.ac.jp